# *ELEMENTS DE CORRIGE*

*Question 1 :*

***a°-***

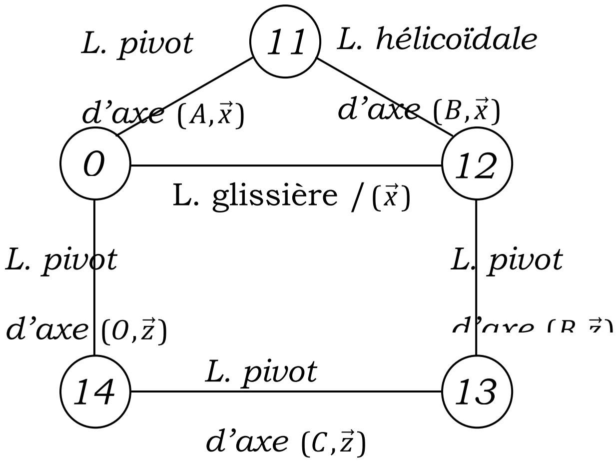
***Variateur - Moteur – Réducteur – Système vis-écrou – Système bielle-manivelle.***

***b°-***

***Carte de commande – Codeur incrémental – Interface H/M***

*Question 2 :*

***a°- Graphe de liaisons :***




***b°- Fermeture cinématique :***

$\vec{V}(C\in 14/0) = \cancel{\vec{V}(C\in 14/13)} + \vec{V}(C\in 13/12) + \vec{V}(C\in 12/0)$

$\dot{\theta}.\left(dy.\vec{x}_{14} + (dx+R).\vec{y}_{14}\right) = L.\dot{\alpha}.\vec{y}_{13} + \dot{x}.\vec{x}$

$/\,\vec{x}$ $\boxed{dy.\dot{\theta}.Cos\theta - (dx+R).\dot{\theta}.Sin\theta = -L.\dot{\alpha}.Sin\alpha + \dot{x}}$

$/\,\vec{y}$ $\boxed{dy.\dot{\theta}.Sin\theta + (dx+R).\dot{\theta}.Cos\theta = L.\dot{\alpha}.Cos\alpha}$

***c°- Fermeture géométrique :***

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC}$

$(dx+R).\vec{x}_{14} - dy.\vec{y}_{14} = -e.\vec{y} + x.\vec{x} + L.L.\vec{x}_{13}$

$/\,\vec{x}$ $\boxed{(dx+R)Cos\theta + dy.Sin\theta = x + L.Cos\alpha}$

$/\,\vec{y}$ $\boxed{(dx+R).Sin\theta - dy.Cos\theta = -e + L.Sin\alpha}$

***d°-*** $\vec{V}(B\in 12/11) = \dfrac{p}{2\pi}.\vec{\Omega}(12/11)$ $\qquad \vec{V}(B\in 12/0) - \vec{V}(B\in 11/0) = \dfrac{p}{2\pi}.\left(\vec{\Omega}(12/0) - \vec{\Omega}(11/0)\right)$

$\dot{x}.\vec{x} = -\dfrac{p}{2\pi}.\omega_{11}.\vec{x}$ $\qquad \dot{x} = -\dfrac{p}{2\pi}.\omega_{11}$ $\qquad \dot{x} = -\dfrac{p}{2\pi}.k.\omega_m$

$$\dot{x} = -\frac{p}{2\pi}.k.\frac{\pi}{30}.N_{mot} = -\frac{5}{2\pi}.\left(\frac{\pi}{30}.\frac{1}{4,3}.6930\right) = -134,3(mm/s)$$

*Question 3 :*

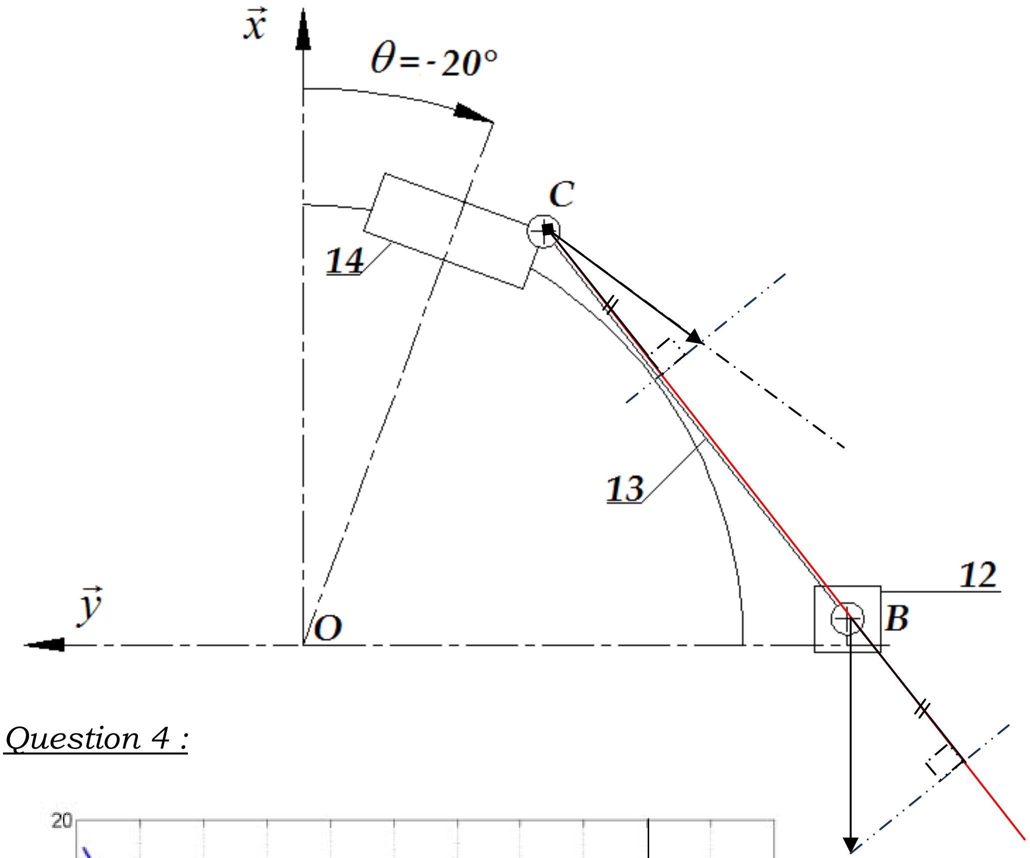
***b°-*** $\vec{V}(C,14/0) = \underbrace{\vec{V}(O,14/0)}_{\vec{0}} + \vec{\Omega}(14/0)\wedge\overrightarrow{OC} \quad \Rightarrow \vec{V}(C,14/0)\perp\overrightarrow{OC}$

***c°-*** $\vec{V}(B,13/0) = \underbrace{\vec{V}(B,13/12)}_{\vec{0}} + \vec{V}(B,12/0)$

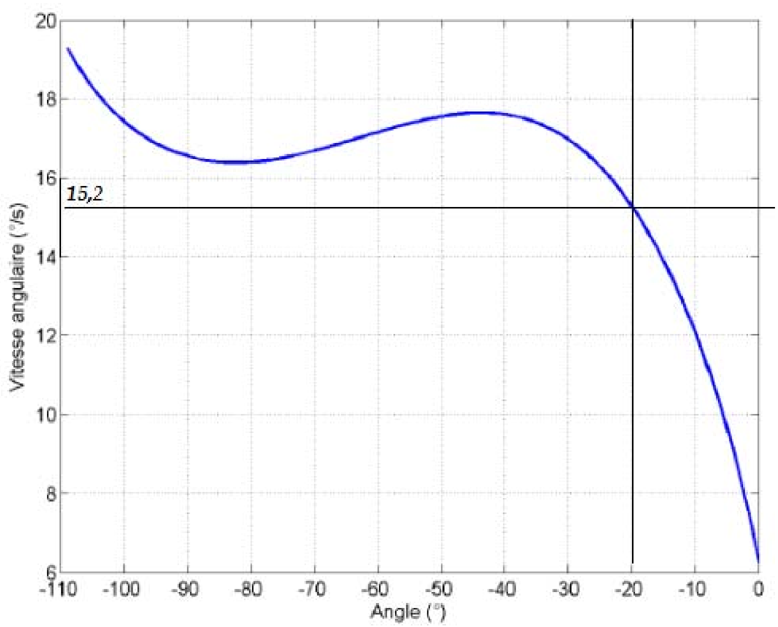
$\vec{V}(C,14/0) = \underbrace{\vec{V}(C,14/13)}_{\vec{0}} + \vec{V}(C,13/0)$

Par équiprojectivité on a : $\vec{V}(C,13/0).\overrightarrow{BC} = \vec{V}(B,13/0).\overrightarrow{BC}$

$\left\|\vec{V}(C,14/0)\right\| = OC.\dot{\theta} \Rightarrow \dot{\theta} = \dfrac{\left\|\vec{V}(C,14/0)\right\|}{\sqrt{(R+dx)^2+dy^2}} \Rightarrow \dot{\theta} = ..........$ $\qquad \sqrt{(R+dx)^2+dy^2} \approx 0,543$




*Question 4 :*




$\dot{\theta}_{th} = \text{.........}(rad/s) \Rightarrow \dot{\theta}_{th} = \text{.........}(°/s)$

$\dot{\theta}_{simulat} \approx 15{,}2(°/s)$

*La cinématique graphique a permis une bonne estimation de la vitesse angulaire.*

*Question 5 :*

**a°-** $T(\Sigma_1/0) = T(Am/0) + T(Réd/0) + T(11/0) + T(12/0)$

$$\boxed{T(\Sigma_1/0) = \frac{1}{2}.J_m.\omega_m^2 + \frac{1}{2}.J_r.\omega_m^2 + \frac{1}{2}.J_{11}.\omega_{11}^2 + \frac{1}{2}.M_{12}.\dot{x}^2}$$

**b°-** $T(13/0) = \frac{1}{2}.(M_{13}.\vec{V}(G_{13}/0).\vec{V}(B,13/0) + \vec{\Omega}(13/0).\vec{\sigma}(B,13/0))$

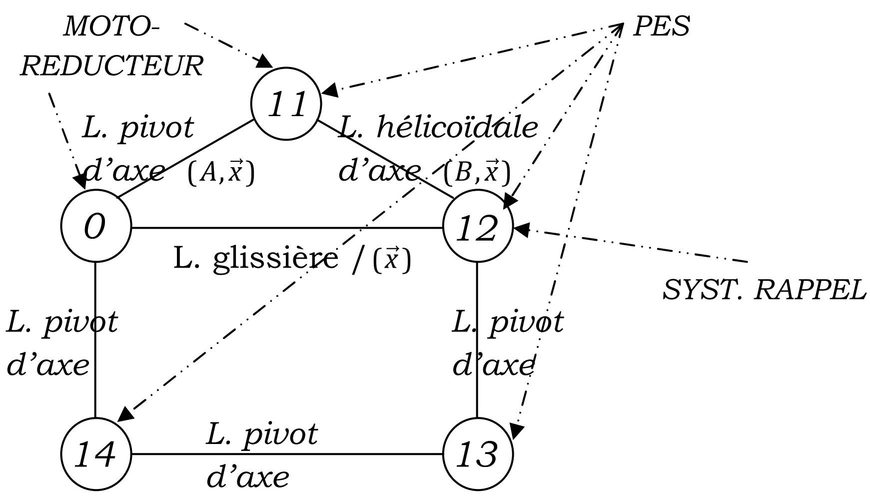
$\vec{V}(B,13/0) = \dot{x}.\vec{x}$ $\quad\quad$ $\vec{V}(G_{13}/0) = \dot{x}.\vec{x} + \frac{L}{2}.\dot{\alpha}.\vec{y}_{13}$ $\quad\quad$ $\vec{\Omega}(13/0) = \dot{\alpha}.\vec{z}$

$$\vec{z}.\vec{\sigma}(B,13/0)=\vec{z}.\left(M_{13}.\overrightarrow{BG_{13}}\wedge\vec{V}(B,13/0)+I(B,13).\vec{\Omega}(13/0)\right)=-M_{13}.\dot{x}.\frac{L}{2}.\dot{\alpha}.Sin\alpha+J_{13}.\dot{\alpha}$$

$$\boxed{T(13/0)=\frac{1}{2}.\left(M_{13}.\dot{x}^2-2.M_{13}.\dot{x}.\frac{L}{2}.\dot{\alpha}.Sin\alpha+J_{13}.\dot{\alpha}^2\right)}$$

***c°-*** $T(14/0)=\frac{1}{2}.J_{O,\vec{z}}(14).\boldsymbol{\omega}^2{}_{14/0}$

$$\boxed{T(14/0)=\frac{1}{2}.(J_{14}+M_{14}.R^2).\dot{\theta}^2}$$




***d°-*** $\boxed{P_{INT}=P_{dissipée}=(\eta_v.\eta_r-1).P_{MOTUR}=(\eta_v.\eta_r-1).C_m.\boldsymbol{\omega}_m}$

***e°-*** $\boxed{P_{EXT}=C_m\boldsymbol{\omega}_m-F_{rappel}.\dot{x}-M_{12}.g.\dot{x}+M_{14}.g.R.\dot{\theta}.Sin\theta-M_{13}.g.\dot{x}+M_{13}.g.\frac{L}{2}.\dot{\alpha}.Sin\alpha}$

- L'axe moteur et la vis à bille sont supposés équilibrés dynamiquement.

*Question 6 :*

***a°-*** $e.\dot{\theta}=\dot{x}$ $\quad dy.\dot{\theta}=-L.\dot{\alpha}$ $\qquad \dot{x}=-\frac{p}{2\pi}.k.\boldsymbol{\omega}_m$ $\qquad \dot{\theta}=-\frac{p}{2\pi}.\frac{k}{e}.\boldsymbol{\omega}_m$ $\qquad \dot{\alpha}=\frac{dy}{L}.\frac{p}{2\pi}.\frac{k}{e}.\boldsymbol{\omega}_m$

$$T(\Sigma/0)=\frac{1}{2}.(J_{14}+M_{14}.R^2).\dot{\theta}^2+\frac{1}{2}.\left(M_{13}.\dot{x}^2-M_{13}.\dot{x}.\frac{L}{2}.\dot{\alpha}.Sin\alpha+J_{13}.\dot{\alpha}^2\right)$$
$$+\frac{1}{2}.(J_m+J_r+J_{11}.k^2+M_{12}.(\frac{p}{2\pi}.k)^2).\boldsymbol{\omega}_m{}^2$$

$$T(\Sigma/0)=\frac{1}{2}.\left(\left((J_{14}+M_{14}.R^2).+J_{13}.\left(\frac{dy}{L}\right)^2\right)\left(\frac{p}{2\pi}.\frac{k}{e}\right)^2+J_m+J_r+J_{11}.k^2+\left(M_{12}+M_{13}\right).(\frac{p}{2\pi}.k)^2\right)\boldsymbol{\omega}_m{}^2$$

$$\boxed{J_{éq}=\left((J_{14}+M_{14}.R^2).+J_{13}.\left(\frac{dy}{L}\right)^2\right)\left(\frac{p}{2\pi}.\frac{k}{e}\right)^2+J_m+J_r+J_{11}.k^2+\left(M_{12}+M_{13}\right).(\frac{p}{2\pi}.k)^2}$$

***b°-*** $\frac{d}{dt}T(\Sigma/0)=P_{EXT}+P_{INT}$

$$J_{éq}.\frac{d}{dt}\omega_m.\omega_m=C_m.\omega_m-F_{rappel}.\dot{x}-M_{12}.g.\dot{x}-M_{14}.g.R.\dot{\theta}-M_{13}.g.\dot{x}+(1-\eta_v.\eta_r).C_m.\omega_m$$

$$J_{éq}.\frac{d}{dt}\omega_m=\eta_v.\eta_r.C_m-F_{rappel}.\frac{\dot{x}}{\omega_m}-M_{12}.g.\frac{\dot{x}}{\omega_m}-M_{14}.g.R.\frac{\dot{\theta}}{\omega_m}-M_{13}.g.\frac{\dot{x}}{\omega_m}$$

$$J_{éq}.\frac{d}{dt}\omega_m=\eta_v.\eta_r.C_m-F_{rappel}.\left(-\frac{p}{2\pi}.k\right)-M_{12}.g.\left(-\frac{p}{2\pi}.k\right)-M_{14}.g.R.\left(-\frac{p}{2\pi}.\frac{k}{e}\right)-M_{13}.g.\left(-\frac{p}{2\pi}.k\right)$$

$$\boxed{C_m=\frac{1}{\eta_v.\eta_r}.\left(J_{éq}.\frac{d}{dt}\omega_m-\left(F_{rappel}+\left(M_{12}+M_{13}+M_{14}.\frac{R}{e}\right).g\right).\frac{p.k}{2\pi}\right)}$$

*<u>Question 7 :</u>*

***a°-*** $\{\mathcal{C}(3/R_0)\}=\left\{\begin{matrix}\vec{R}_c(3/R_0)\\ \vec{\sigma}(O,3/R_0)\end{matrix}\right\}_O$ $\left(\frac{d}{dt}\vec{x}_3\right)_0=\dot{\theta}_3.\vec{y}_3+\dot{\theta}_2.Sin\theta_3.\vec{z}_3$

$$\vec{R}_c(3/R_0)=M_3.\vec{V}(G_3\in 3/R_0)=M_3.R.\left(\dot{\theta}_3.\vec{y}_3+\dot{\theta}_2.Sin\theta_3.\vec{z}_3\right)$$

$$\vec{\sigma}(O,3/R_0)=M_3.\overrightarrow{OG_3}\wedge\vec{V}(O\in 3/R_0)+\bar{\bar{I}}(O,3).\vec{\Omega}(3/0)$$

$$=\begin{pmatrix}A_3&0&0\\0&B_3&0\\0&0&C_3\end{pmatrix}_{(\vec{x}_3,\vec{y}_3,\vec{z}_3)}\begin{pmatrix}\dot{\theta}_2.Cos\theta_3\\-\dot{\theta}_2.Sin\theta_3\\\dot{\theta}_3\end{pmatrix}_{(\vec{x}_3,\vec{y}_3,\vec{z}_3)}\qquad \vec{\sigma}(O,3/R_0)=\begin{pmatrix}A_3.\dot{\theta}_2.Cos\theta_3\\-B_3.\dot{\theta}_2.Sin\theta_3\\C_3.\dot{\theta}_3\end{pmatrix}_{(\vec{x}_3,\vec{y}_3,\vec{z}_3)}$$

$$\boxed{\{\mathcal{C}(3/R_0)\}=\left\{\begin{matrix}\vec{R}_c(3/R_0)\\ \vec{\sigma}(O,3/R_0)\end{matrix}\right\}_O=\left\{\begin{matrix}M_3.R.\left(\dot{\theta}_3.\vec{y}_3+\dot{\theta}_2.Sin\theta_3.\vec{z}_3\right)\\ A_3.\dot{\theta}_2.Cos\theta_3.\vec{x}_3-B_3.\dot{\theta}_2.Sin\theta_3.\vec{y}_3+C_3.\dot{\theta}_3.\vec{z}_3\end{matrix}\right\}_O}$$

***b°-*** $\{\mathcal{C}(4/R_0)\}=\left\{\begin{matrix}\vec{R}_c(4/R_0)\\ \vec{\sigma}(O,4/R_0)\end{matrix}\right\}_O$ $\vec{R}_c(4/R_0)=M_4.\vec{V}(E\in 4/R_0)=M_4.\left(\dot{\lambda}.\vec{x}_3+\lambda.\left(\dot{\theta}_3.\vec{y}_3+\dot{\theta}_2.Sin\theta_3.\vec{z}_3\right)\right)$

$$\vec{\sigma}(E,4/R_0)=\bar{\bar{I}}(E,4).\vec{\Omega}(4/0)$$

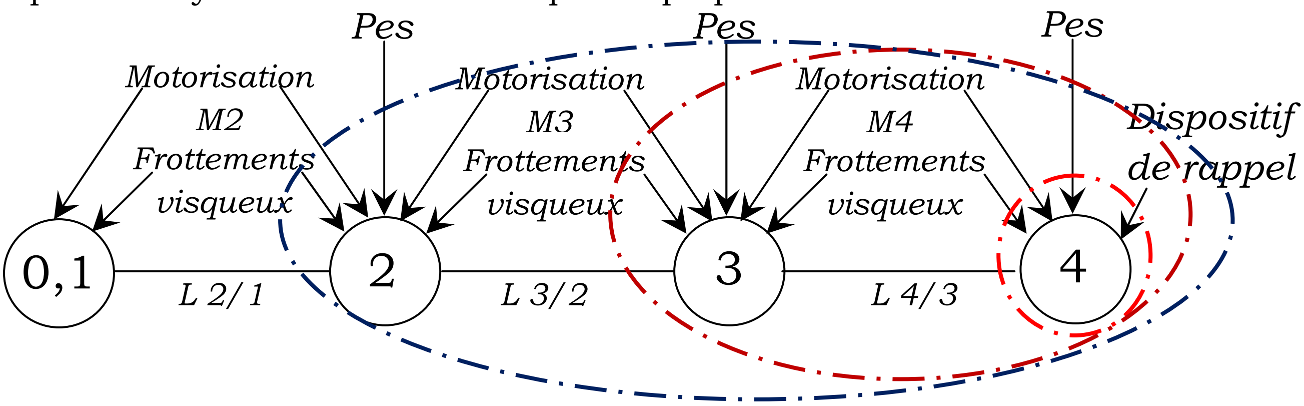
$$=\begin{pmatrix}A_4&0&0\\0&B_4&0\\0&0&C_4\end{pmatrix}_{(\vec{x}_3,\vec{y}_3,\vec{z}_3)}\begin{pmatrix}\dot{\theta}_2.Cos\theta_3\\-\dot{\theta}_2.Sin\theta_3\\\dot{\theta}_3\end{pmatrix}_{(\vec{x}_3,\vec{y}_3,\vec{z}_3)}\qquad \vec{\sigma}(E,4/R_0)=\begin{pmatrix}A_4.\dot{\theta}_2.Cos\theta_3\\-B_4.\dot{\theta}_2.Sin\theta_3\\C_4.\dot{\theta}_3\end{pmatrix}_{(\vec{x}_3,\vec{y}_3,\vec{z}_3)}$$

$$\vec{\sigma}(O,4/R_0)=\vec{\sigma}(E,4/R_0)+M_4.\vec{V}(E\in 4/R_0)\wedge\overrightarrow{EO}\qquad \vec{\sigma}(O,4/R_0)=\begin{pmatrix}A_4.\dot{\theta}_2.Cos\theta_3\\-\left(B_4+M_4.\lambda^2\right).\dot{\theta}_2.Sin\theta_3\\\left(C_4+M_4.\lambda^2\right).\dot{\theta}_3\end{pmatrix}_{(\vec{x}_3,\vec{y}_3,\vec{z}_3)}$$

$$\boxed{\{\mathcal{C}(4/R_0)\}=\left\{\begin{matrix}\vec{R}_c(4/R_0)\\ \vec{\sigma}(O,4/R_0)\end{matrix}\right\}_O=\left\{\begin{matrix}M_4.\left(\dot{\lambda}.\vec{x}_3+\lambda.\left(\dot{\theta}_3.\vec{y}_3+\dot{\theta}_2.Sin\theta_3.\vec{z}_3\right)\right)\\ A_4.\dot{\theta}_2.Cos\theta_3.\vec{x}_3-\left(B_4+M_4.\lambda^2\right).\dot{\theta}_2.Sin\theta_3.\vec{y}_3+\left(C_4+M_4.\lambda^2\right).\dot{\theta}_3.\vec{z}_3\end{matrix}\right\}_O}$$

*Question 8 :*

Le graphe d'analyse du modèle mécanique est proposé ci-dessous :




| Effort | Ensemble isolé | Théorème utilisé | Justification du choix d'isolement et de théorème |
|---|---|---|---|
| $C_{M2}$ | 2 , 3 , 4 | *T.M.D en O en projection sur* $\vec{x}_2$ | 3, 4 sont entrainés par 2<br>Eviter les inconnues statiques de L2/1 |
| $C_{M3}$ | 3 , 4 | *T.M.D en O en projection sur* $\vec{z}_2$ | 4 est entrainé par 3<br>Eviter les inconnues statiques de L3/2 |
| $F_{M4}$ | 4 | *T.R.D en projection sur* $\vec{x}_3$ | Aucun solide entrainé par 4<br>Eviter les inconnues statiques de L4/3 |

*Question 9 :*

*On isole* $(4)$ et on applique le *T.R.D. en projection sur* $\vec{x}_3$ :

$$\vec{x}_3.\vec{R}\left(\overline{4}\rightarrow 4\right)=\vec{x}_3.M_4.\vec{a}\left(E,4/0\right)$$

$$\vec{x}_3.\vec{R}\left(3\xrightarrow{mot}4\right)+\vec{x}_3.\vec{R}\left(3\xrightarrow{L}4\right)+\vec{x}_3.\vec{R}\left(3\xrightarrow{frott}4\right)+\vec{x}_3.\vec{R}\left(pes\rightarrow 4\right)+\vec{x}_3.\vec{R}\left(DR\rightarrow 4\right)$$

$$=M_4.\left(\frac{d}{dt}\vec{x}_3.\vec{V}\left(E,4/0\right)-\left[\frac{d}{dt}\vec{x}_3\right]_0.\vec{V}\left(E,4/0\right)\right)$$

$$F_{M4}-f_4.\dot{\lambda}-M_4.g.\vec{x}.\vec{x}_3-F_R=M_4.\ddot{\lambda}-\left(\frac{d}{dt}\vec{x}_3\right)_0.M_4.\left(\dot{\lambda}.\vec{x}_3+\lambda.\left(\dot{\theta}_3.\vec{y}_3+\dot{\theta}_2.Sin\theta_3.\vec{z}_3\right)\right)$$

$$\vec{x}.\vec{x}_3=\begin{pmatrix}Cos\theta_1\\-Sin\theta_1.Cos\theta_2\\Sin\theta_1.Sin\theta_2\end{pmatrix}_{B2}\begin{pmatrix}Cos\theta_3\\Sin\theta_3\\0\end{pmatrix}_{B2}=Cos\theta_1.Cos\theta_3-Sin\theta_1.Cos\theta_2.Sin\theta_3 \qquad \left(\frac{d}{dt}\vec{x}_3\right)_0=\dot{\theta}_3.\vec{y}_3+\dot{\theta}_2.Sin\theta_3.\vec{z}_3$$

$$\boxed{F_{M4}-f_4.\dot{\lambda}-M_4.g.\left(Cos\theta_1.Cos\theta_3-Sin\theta_1.Cos\theta_2.Sin\theta_3\right)-F_R=M_4.\left(\ddot{\lambda}-\lambda.\left(\dot{\theta}_3^{\,2}+\dot{\theta}_2^{\,2}.Sin^2\theta_3\right)\right)}$$

*Question 10 :*

*On isole* $E_{34}$ et on applique le *T.M.D. en O en projection /* $\vec{z}_2$

$$\vec{z}_2.\vec{M}(0,\bar{E}_{34} \to E_{34}) = \vec{z}_2.\vec{\delta}(0,E_{34}/R_0)$$

$$\vec{z}_2.\vec{M}(0,\bar{E}_{34} \to E_{34}) = \underbrace{\vec{z}_2.\vec{M}(0,2 \xrightarrow{L} 3)}_{0} + \underbrace{\vec{z}_2.\vec{M}(0,2 \xrightarrow{frott} 3)}_{-f_3.\dot{\theta}_3} + \underbrace{\vec{z}_2.\vec{M}(0,2 \xrightarrow{mot} 3)}_{C_{M3}} + \vec{z}_2.\vec{M}(0,pes \to E_{34}) + \underbrace{\vec{z}_2.\vec{M}(0,DR \to 4)}_{0}$$

$$\vec{z}_2.\vec{M}(0,pes \to E_3) = \vec{z}_2.\left(\overrightarrow{OG}_{34} \wedge -M_{34}.g.\vec{x}\right) = M_{34}.g.R_{34}.\left(Cos\theta_1.Sin\theta_3 + Sin\theta_1.Cos\theta_2.Cos\theta_3\right)$$

$$\vec{z}_2.\vec{\delta}(0,E_{34}/R_0) = \frac{d}{dt}\vec{z}_2.\vec{\sigma}(0,E_{34}/R_0) - \left[\frac{d\vec{z}_2}{dt}\right]_{R_0}.\vec{\sigma}(0,E_{34}/R_0) \qquad \left(\frac{d}{dt}\vec{z}_2\right)_0 = -\dot{\theta}_2.\vec{y}_2 = -\dot{\theta}_2\begin{pmatrix} Sin\theta_3 \\ Cos\theta_3 \\ 0 \end{pmatrix}_{B3}$$

$$\vec{\sigma}(O,E_{34}/R_0) = \begin{pmatrix} A_{34}.\dot{\theta}_2.Cos\theta_3 \\ -B_{34}.\dot{\theta}_2.Sin\theta_3 \\ C_{34}.\dot{\theta}_3 \end{pmatrix}_{(\vec{x}_3,\vec{y}_3,\vec{z}_3)} \qquad \vec{z}_2.\vec{\delta}(0,E_{34}/R_0) = \frac{d}{dt}\vec{z}_2.\vec{\sigma}(0,E_{34}/R_0) - \left[\frac{d}{dt}\vec{z}_2\right]_{R_0}.\vec{\sigma}(0,E_{34}/R_0)$$

$$\boxed{C_{M3} - f_3.\dot{\theta}_3 + M_{34}.R_{34}.g.\left(Cos\theta_1.Sin\theta_3 + Sin\theta_1.Cos\theta_2.Cos\theta_3\right) = C_{34}.\ddot{\theta}_3 + \left(B_{34} - A_{34}\right).\dot{\theta}_2^2.Sin\theta_3.Cos\theta_3}$$

*Question 11 :*

***a°-*** *On isole l'ensemble* $E_2 = \left(2+3+4\right)$, *on applique le T.M.S en O en projection sur* $\vec{x}_1$ :

$$\vec{x}_1.\vec{M}(0,\bar{E}_2 \to E_2) = 0$$

$$\vec{x}_1.\vec{M}(0,\bar{E}_2 \to E_2) = \underbrace{\vec{x}_1.\vec{M}(0,1 \xrightarrow{L} 2)}_{0} + \underbrace{\vec{x}_1.\vec{M}(0,1 \xrightarrow{frein} 2)}_{C_{f2}} + \vec{x}_1.\vec{M}(0,pes \to E_2) + \underbrace{\vec{x}_1.\vec{M}(0,DR \to 4)}_{0}$$

$$\vec{x}_1.\vec{M}(0,pes \to 3) = \vec{x}_1.\left(\overrightarrow{OG}_3 \wedge -M_3.g.\vec{x}\right) = -M_3.g.R.\vec{x}_3.\left(\vec{x} \wedge \vec{x}_1\right) = -M_3.g.R.\vec{x}_3.\left(Sin\theta_1.\vec{z}_1\right) = -M_3.g.R.Sin\theta_1.Sin\theta_2.Sin\theta_3$$

$$\boxed{C_{f2} - \left(M_3.R + M_4.\lambda_{\max}\right).g.Sin\theta_1.Sin\theta_2.Sin\theta_3 = 0}$$

***b°-*** $\theta_1 = -90° \quad \theta_2 = -90° \quad \theta_3 = -90°$ $\qquad \boxed{C_{f2} = -\left(M_3.R + M_4.\lambda_{MAX}\right).g}$

*Question 12 :*

$$C_{f2} = \frac{\eta.C_{frein}}{\rho} \Rightarrow \qquad \boxed{C_{frein} = \frac{\rho.C_{f2}}{\eta}}$$

*Question 13 :*

***a°-*** $\vec{V}(M \in Wg/S) = \vec{V}(O \in Wg/S) + \vec{\Omega}(Wg/S) \wedge \overrightarrow{OM} = r.w_g.\vec{v}$ $\qquad \vec{t}_M(S \to Wg) = -f.P.\vec{v}$

***b°-*** $\vec{x}_2.\vec{M}_O(S \to Wg) = \vec{x}_2.\int_M \overrightarrow{OM} \wedge \vec{f}_M(S \to Wg).ds = \vec{x}_2.\int r.\vec{u} \wedge \left(-P.\vec{x}_2 - f.P.\vec{v}\right)ds$

$$\vec{x}_2.\vec{M}_O(S \rightarrow Wg) = -f.P.\int_{R_i}^{R_e} r^2.dr.\int_0^{2\pi} d\theta \qquad \vec{x}_2.\vec{M}_O(S \rightarrow Wg) = -\frac{2}{3}.f.P.\pi\left(R_e^3 - R_i^3\right)$$

$$\boxed{C_{f2} = \frac{2}{3}.f.P.\pi\left(R_e^3 - R_i^3\right)}$$

*Question 14 :*

*Réducteur à tarin épicycloïdal :*

$$\frac{\boldsymbol{\omega_{As}} - \boldsymbol{\omega_{Am}}}{\boldsymbol{\omega_0} - \boldsymbol{\omega_{Am}}} = (-1)^0.\frac{\boldsymbol{Z_0}}{\boldsymbol{Z_{2a}}}.\frac{\boldsymbol{Z_{2b}}}{\boldsymbol{Z_1}} \Rightarrow -\frac{\boldsymbol{\omega_{As}}}{\boldsymbol{\omega_{Am}}} + 1 = \frac{\boldsymbol{Z_0}}{\boldsymbol{Z_{2a}}}.\frac{\boldsymbol{Z_{2b}}}{\boldsymbol{Z_1}} \Rightarrow \frac{\boldsymbol{\omega_{As}}}{\boldsymbol{\omega_{Am}}} = 1 - \frac{\boldsymbol{Z_0}}{\boldsymbol{Z_{2a}}}.\frac{\boldsymbol{Z_{2b}}}{\boldsymbol{Z_1}}$$

$$\Rightarrow \frac{\boldsymbol{\omega_{As}}}{\boldsymbol{\omega_{Am}}} = \frac{\boldsymbol{Z_{2a}.Z_1 - Z_0.Z_{2b}}}{\boldsymbol{Z_{2a}.Z_1}}$$

$$\boxed{\frac{\boldsymbol{\omega_{As}}}{\boldsymbol{\omega_{Am}}} = \frac{\boldsymbol{Z_{2a}.Z_1 - Z_0.Z_{2b}}}{\boldsymbol{Z_{2a}.Z_1}}} \qquad \frac{\boldsymbol{\omega_{As}}}{\boldsymbol{\omega_{Am}}} = \frac{20.100 - 66.30}{20.100} = \frac{1}{100}$$

*Question 15 :*

*Etude cinématique du réducteur* Harmonic Drive*:*

$$\vec{V}(I \in Fs/0) = \vec{V}(H \in Fs/0) + \vec{\Omega}(Fs/0) \wedge \overrightarrow{HI}$$

$$\vec{V}(H \in Fs/0) = \vec{V}(H \in Fs/Wg) + \vec{V}(H \in Wg/0) = \vec{V}(P \in Wg/0) + \vec{\Omega}(Wg/0) \wedge \overrightarrow{PH} = \omega_{Wg}.r.\vec{z}_{Wg}$$

$$\vec{V}(I \in Fs/0) = \omega_{Wg}.r.\vec{z}_{Wg} + \omega_{FS}.R_{FS}.\vec{z}_{Wg}$$

$$R.S.G. \Rightarrow \omega_{Wg}.r = -\omega_{FS}.R_F \qquad \Rightarrow \omega_{Wg}.\left(R_C - R_F\right) = -\omega_{FS}.R_F \qquad \Rightarrow \frac{\omega_{FS}}{\omega_{Wg}} = -\frac{R_C - R_F}{R_F}$$

$$\Rightarrow \frac{\omega_{FS}}{\omega_{Wg}} = -\frac{Z_{CS} - Z_{FS}}{Z_{FS}} = -\frac{1}{100} \Rightarrow \begin{matrix} Z_{CS} = 202 dents \\ Z_{FS} = 200 dents \end{matrix}$$

*Expliquer l'intérêt de ce type de réducteur :*

*Rapport de réduction important, faible encombrement, rendement élevé*

# *Partie III : Analyse de la régulation en effort de l'axe 4 :*

## *Question 16 :*

***a°-*** $\Omega_{red}(p) = A_m(p).U_m(p) - A_F(p).F_r(p)$

$$\Omega_{red}(p) = \frac{\dfrac{k}{R+L.p}.\dfrac{1}{f+I_m.p}}{1+\dfrac{k}{R+L.p}.\dfrac{1}{f+I_m.p}}.K_{red}.U_m(p) - \frac{\dfrac{1}{f+I_m.p}.K_{red}.R_p}{1+\dfrac{k}{R+L.p}.\dfrac{1}{f+I_m.p}}.K_{red}.F_r(p)$$

$$\Omega_{red}(p) = \frac{\dfrac{k.K_{red}}{k^2+R.f}}{1+\dfrac{R.I_m+f.L}{k^2+R.f}.p+\dfrac{L.I_m}{k^2+R.f}p^2}.U_m(p) - \frac{\dfrac{R.K_{red}{}^2.R_P}{k^2+R.f}.\left(1+\dfrac{L}{R}.p\right)}{1+\dfrac{R.I_m+f.L}{k^2+R.f}.p+\dfrac{L.I_m}{k^2+R.f}p^2}.F_r(p)$$

$$\boxed{A_m(p) = \frac{\dfrac{k.K_{red}}{k^2+R.f}}{1+\dfrac{R.I_m+f.L}{k^2+R.f}.p+\dfrac{L.I_m}{k^2+R.f}p^2}} \qquad \boxed{A_F(p) = \frac{\dfrac{R.K_{red}{}^2.R_P}{k^2+R.f}.\left(1+\dfrac{L}{R}.p\right)}{1+\dfrac{R.I_m+f.L}{k^2+R.f}.p+\dfrac{L.I_m}{k^2+R.f}p^2}}$$

$$\boxed{K_m = \frac{K.K_{red}}{k^2+R.f}} \qquad \frac{1}{\omega_0{}^2} = \frac{L.I_m}{k^2+R.f} \Rightarrow \boxed{\omega_0 = \sqrt{\frac{k^2+R.f}{L.I_m}}} \qquad \boxed{\tau = \frac{L}{R}}$$

$$\boxed{K_r = \frac{R.K_{red}{}^2.R_P}{k^2+R.f}} \qquad \frac{2.z}{\omega_0} = \frac{R.I_m+f.L}{k^2+R.f} \Rightarrow \boxed{z = \frac{R.I_m+f.L}{2}\sqrt{\frac{1}{L.I_m.\left(k^2+R.f\right)}}}$$

***b°-*** $\omega_{réd} = \lim\limits_{t\to+\infty} \omega_{Réd}(t) = \lim\limits_{p\to 0} p.\Omega_{Réd}(p) = K_m.U_0 - K_r.F_{r0}$

## *Question 17 :*

*On donne ci-contre la réponse indicielle du moteur réducteur pour un échelon de tension d'amplitude* $U_0 = 12(V)$ *pour un effort de traction supposé nul,* $F_r = 0$.

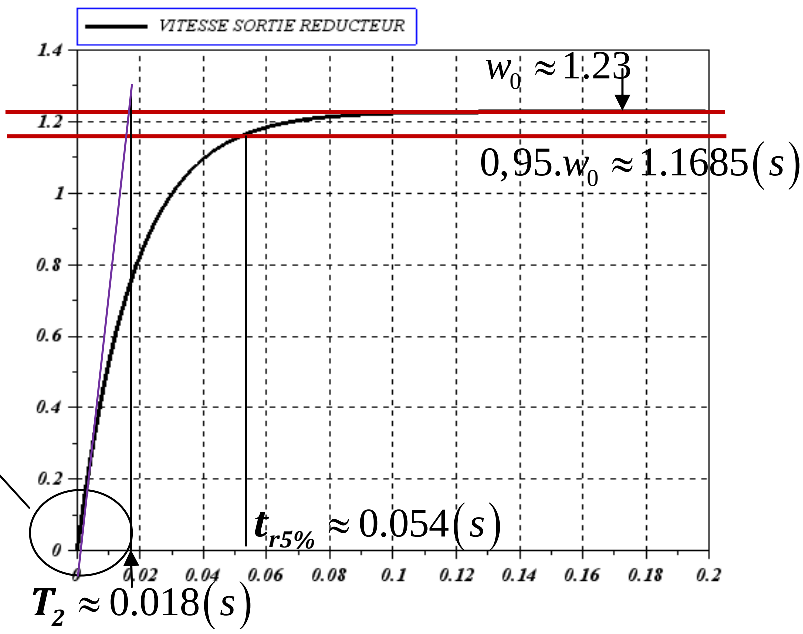
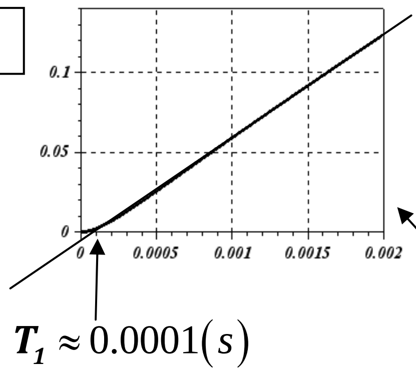



Figure R1

Zoom à l'origine

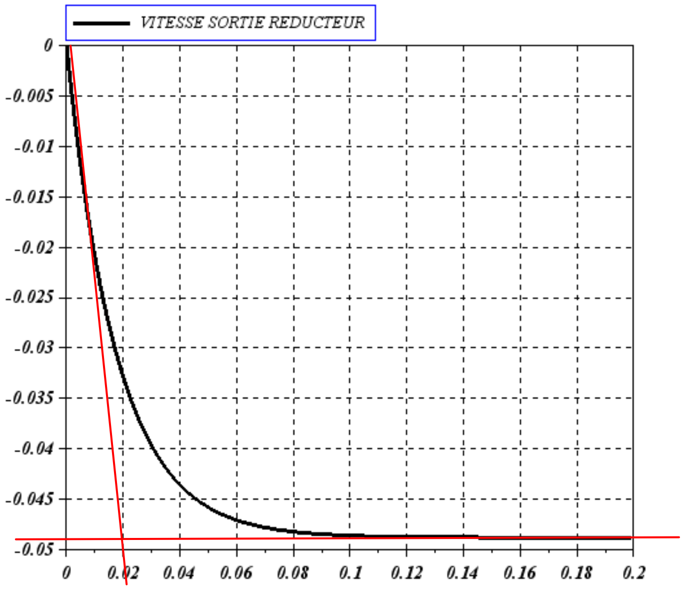


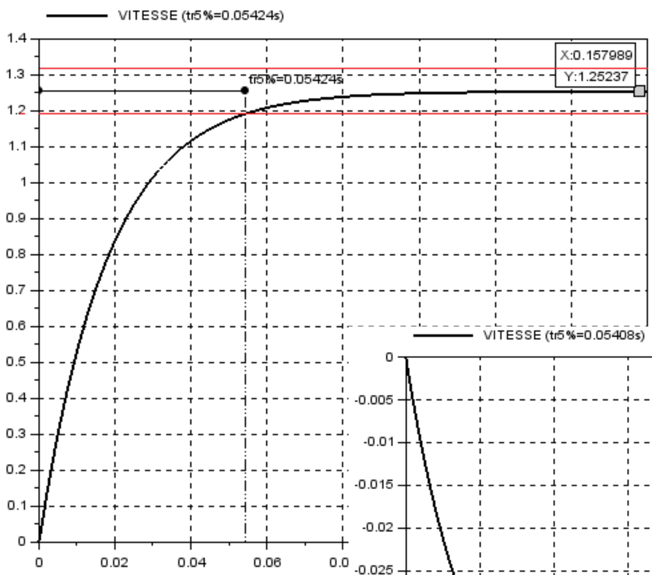
| | |
|---|---|
| *Question 17-a° :* | 2nd ordre Régime apériodique $z>1$ $\boldsymbol{A_m(p)=\dfrac{K_M}{(1+T_1.p).(1+T_2.p)}}$ |
| *Question 17-b° :* | Graphiquement $\boldsymbol{T_1}\approx 0.0001(s)$ . $\boldsymbol{T_2}\approx 0.018(s)$ en supposant $\boldsymbol{T_1}$ négligeable $\boldsymbol{A_m(p)}$ peut être supposé une fonction de transfert du 1er ordre |
| *Question 17-c° :* | $\boldsymbol{T_M=T_2}\approx 0.018(s)$ $\boldsymbol{K_M}\approx\dfrac{1,23}{12}\approx 0,1025\left(\dfrac{rad/s}{V}\right)$ |

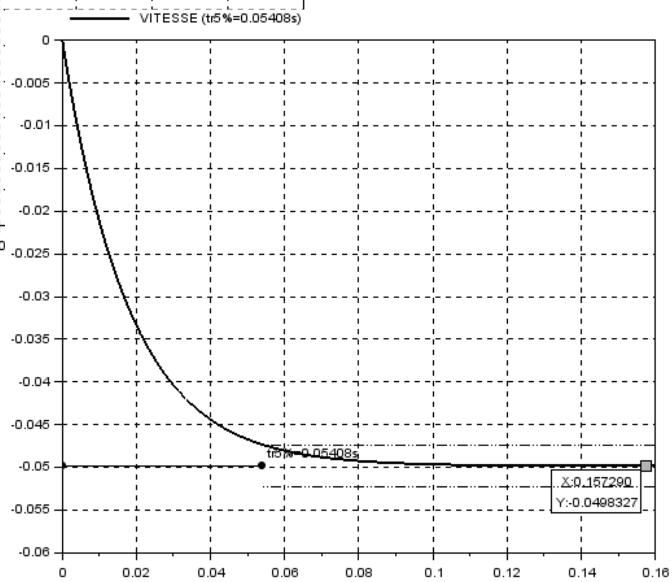
*Question 18 :*

$\boldsymbol{T_M}=0.018(s)$

$$\boldsymbol{K_R}=\frac{0,049}{53}=9,4.10^{-4}\left(\frac{rad/s}{N}\right)$$




*Question 19 :*

$\boldsymbol{a}$°- $\Omega_{red}(p)=B_m(p).U(p)-B_F(p).F_r(p)$

$$B_m(p)=\frac{K_A.A_m(p)}{1+K_A.A_m(p).K_{mot}}=\frac{\boldsymbol{K_A.\dfrac{K_M}{1+T_M.p}}}{\boldsymbol{1+K_A.\dfrac{K_M}{1+T_M.p}.K_{Mot}}}=\frac{\boldsymbol{\dfrac{K_A.K_M}{1+K_A.K_M.K_{Mot}}}}{\boldsymbol{1+\dfrac{T_M}{1+K_A.K_M.K_{Mot}}.p}}$$

$$\boxed{\boldsymbol{G_M=\frac{K_A.K_M}{1+K_A.K_M.K_{Mot}}}} \qquad \boxed{\boldsymbol{T=\frac{T_M}{1+K_A.K_M.K_{Mot}}}}$$

$$B_F(p)=\frac{A_F(p)}{1+K_A.A_m(p).K_{mot}}=\frac{\dfrac{K_R}{1+T_M.p}}{1+K_A.\dfrac{K_M}{1+T_M.p}.K_{Mot}}=\frac{\dfrac{K_R}{1+K_A.K_M.K_{Mot}}}{1+\dfrac{T_M}{1+K_A.K_M.K_{Mot}}.p}$$

$$\boxed{G_R=\frac{K_R}{1+K_A.K_M.K_{Mot}}} \qquad \boxed{T=\frac{T_M}{1+K_A.K_M.K_{Mot}}}$$

**b°-** $\omega_{réd}=\dfrac{K_A.K_M}{1+K_A.K_M.K_{Mot}}.U_0-\dfrac{K_R}{1+K_A.K_M.K_{Mot}}.F_{r0}$

Constante du temps : $T=\dfrac{T_M}{1+K_A.K_M.K_{Mot}}$

L'intérêt de la boucle de vitesse est

- d'améliorer la rapidité du moteur $T<T_M$
- de diminuer la sensibilité du moteur à la perturbation

$$\Delta\omega_{réd}=\omega_{réd}\big|_{sansF_{r0}}-\omega_{réd}\big|_{avecF_{r0}}=\frac{K_R}{1+K_A.K_M.K_{Mot}}.F_{r0}\,.$$

**c°-**

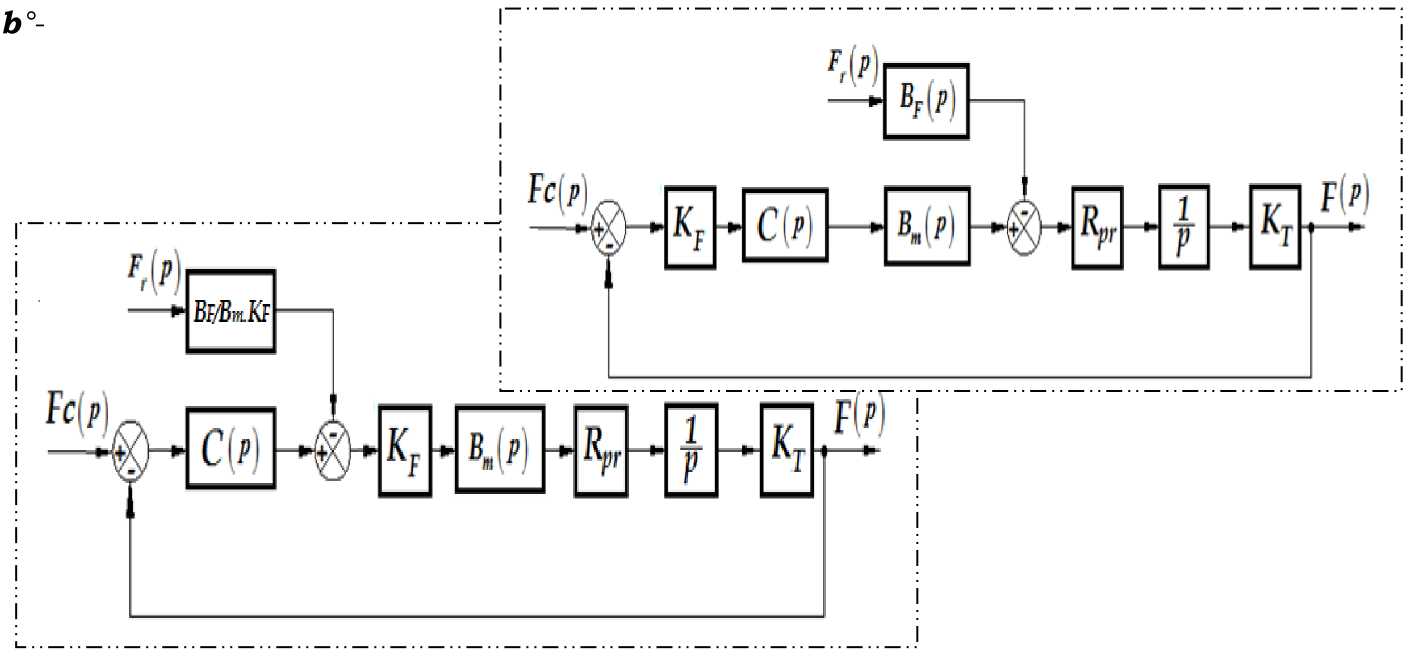
$$B_M(p)=\frac{G_M}{1+T.p}\ ;\quad G_M=0{,}086\left(rad.s^{-1}/V\right)\qquad T=0{,}015(s)$$

$$B_F(p)=\frac{G_R}{1+T.p}\ ;\quad G_R=7{,}8.10^{-4}\left(rad.s^{-1}/N\right)$$

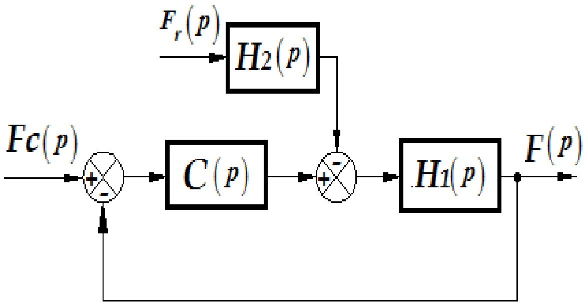
*Question 20 :*

**a°-** $Ecart=K_C.Fc-K_F.F$ *.On espère avoir, en régime permanent, un écart nul lorsque la consigne* $Fc$ *est égale à la sortie* $F$ *.*

**b°-**



***c***°-




$$H_2(p) = K_2 = \frac{G_R}{G_M.K_F}$$

$$H_1(p) = \frac{K_1}{p(1+T.p)} = \frac{K_F.G_M.K_T.R_{pr}}{p(1+T.p)}$$

$$K_1 = K_F.G_M.K_T.R_{pr} = 1*0,086*25 = 2,15$$

*Question 21 :* $H_{BO}(p) = C(p).H_1(p) = \dfrac{C.K_1}{p.(1+T.p)}$ $\quad K_{BO} = C.K_1$ *Classe 1 Ordre 2.*

*Question 22 :* $H_{BF}(p) = \dfrac{H_{BO}(p)}{1+H_{BO}(p)} = \dfrac{1}{1+\dfrac{p}{C.K_1}+\dfrac{T}{C.K_1}.p^2}$

$$\omega_0 = \sqrt{\frac{C.K_1}{T}} \quad \frac{2.\xi}{\omega_0} = \frac{1}{C.K_1} \Rightarrow \xi = \frac{1}{2.C.K_1}.\sqrt{\frac{C.K_1}{T}} \Rightarrow \xi = \frac{1}{2.\sqrt{T.C.K_1}} \Rightarrow \xi^2 = \frac{1}{4.T.C.K_1} \Rightarrow C = \frac{1}{4.T.\xi^2.K_1}$$

*Le système le plus rapide sans dépassement* $\Rightarrow \xi = 1 \Rightarrow C_{tr5\%} = \dfrac{1}{4.T.K_1}$

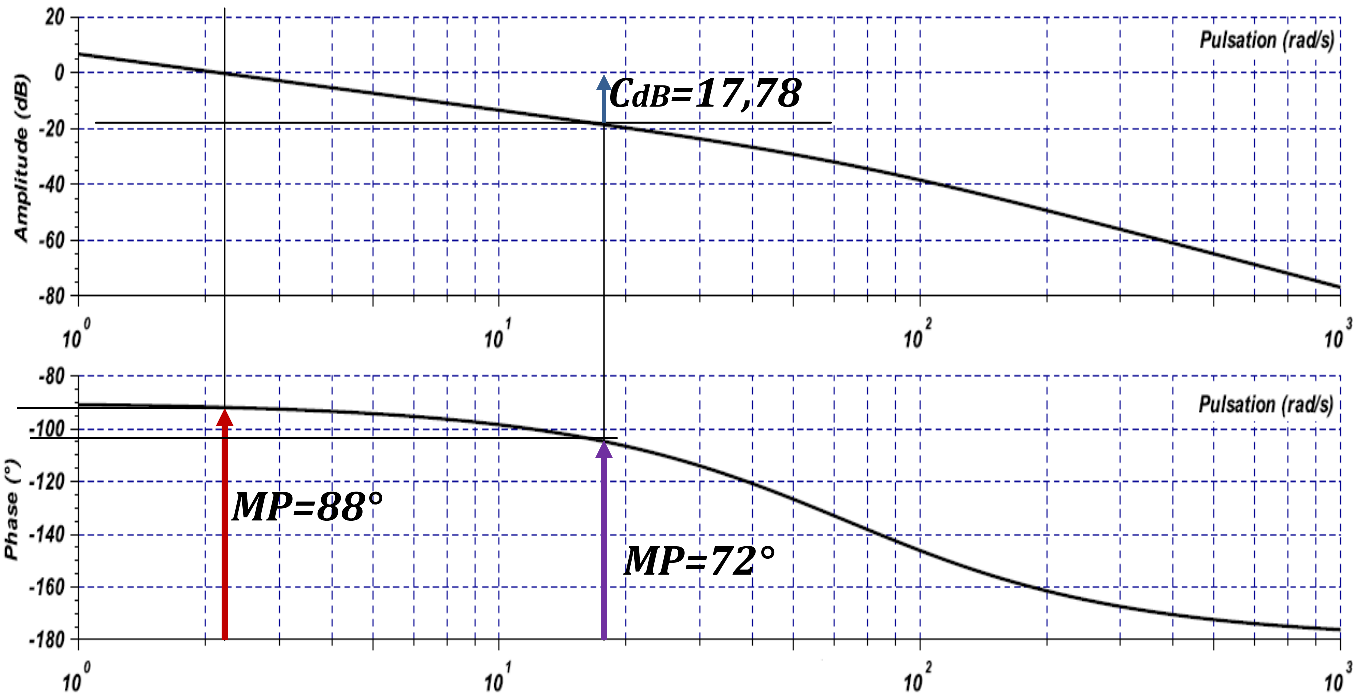
$$C_{tr5\%} = \frac{1}{4.T.K_1} = \frac{1}{4.0,015.2,15} = 7,75$$

*Question 23 :*

***a***°- *Pour* ***C = 1*** ; Marge de phase : $MP = 88°$

Marge de gain : $MG = +\infty$

***b***°- *Pour* $C_{tr5\%} = 7,75 \Rightarrow 20LogC_{tr5\%} = 17,78dB$ ; Marge de phase : $MP = 72°$



*Question 24 :*

**a**°- $H_{BO}(p)$ *de Classe 1* $\Rightarrow \varepsilon_{consigne} = 0$

**b**°- $\Rightarrow \varepsilon_{pert} = \lim_{t\to+\infty} \varepsilon(t)\Big|_{Fc=0} = \lim_{p\to 0} p.\varepsilon(p)\Big|_{Fc=0} = \lim_{p\to 0} p.\dfrac{H_1(p).H_2(p).F_r(p)}{1+H_{BO}(p)} = \lim_{p\to 0} p.\dfrac{\dfrac{K_1}{p.(1+T.p)}.K_2.\dfrac{F_{r0}}{p}}{1+\dfrac{C.K_1}{p.(1+T.p)}}$.

$\Rightarrow \varepsilon_{pert} = \dfrac{K_2}{C}.F_{r0}$

*Question 25 :*

*Exigence de précision non respectée.*

*Question 26 :*

$H_{BO}(p) = C(p).H_1(p) = C.\dfrac{1+T_i.p}{T_i.p}.\dfrac{K_1}{p(1+T.p)}$ $\quad H_{BF}(p) = \dfrac{H_{BO}(p)}{1+H_{BO}(p)} = \dfrac{C.\dfrac{1+T_i.p}{T_i.p}.\dfrac{K_1}{p(1+T.p)}}{1+C.\dfrac{1+T_i.p}{T_i.p}.\dfrac{K_1}{p(1+T.p)}}$

$= \dfrac{C.K_1.(1+T_i.p)}{T_i.p^2.(1+T.p)+C.K_1.(1+T_i.p)}$ $\quad H_{BF}(p) = \dfrac{C.K_1.(1+T_i.p)}{C.K_1 + C.K_1.T_i.p + T_i.p^2 + T_i.T.p^3}$

$1^{er}$ *condition :* $C > 0$ ; $T_i > 0$

$-\begin{vmatrix} T_i.T & C.K_1.T_i \\ T_i & C.K_1 \end{vmatrix} = C.K_1.T_i^2 - T_i.T.C.K_1 > 0 \Rightarrow T_i - T > 0$

$2^{ième}$ *condition :* $\Rightarrow T_i > T$

*Question 27 :*

**a**°- On choisit $\dfrac{1}{T_i} = \dfrac{\omega_C}{10}$ où $\omega_C$ est la pulsation de coupure à 0 dB de la FTBO corrigée par le correcteur proportionnel intégral.

$H_{BO}(p) = C(p).H_1(p) = C.\dfrac{1+T_i.p}{T_i.p}.\dfrac{K_1}{p.(1+T.p)}$

$M\varphi = 180° + \varphi(\omega_C)$ $\quad \varphi(\omega_C) = -180° + \arctan(T_i.\omega_C) - \arctan(T.\omega_C) = -120°$

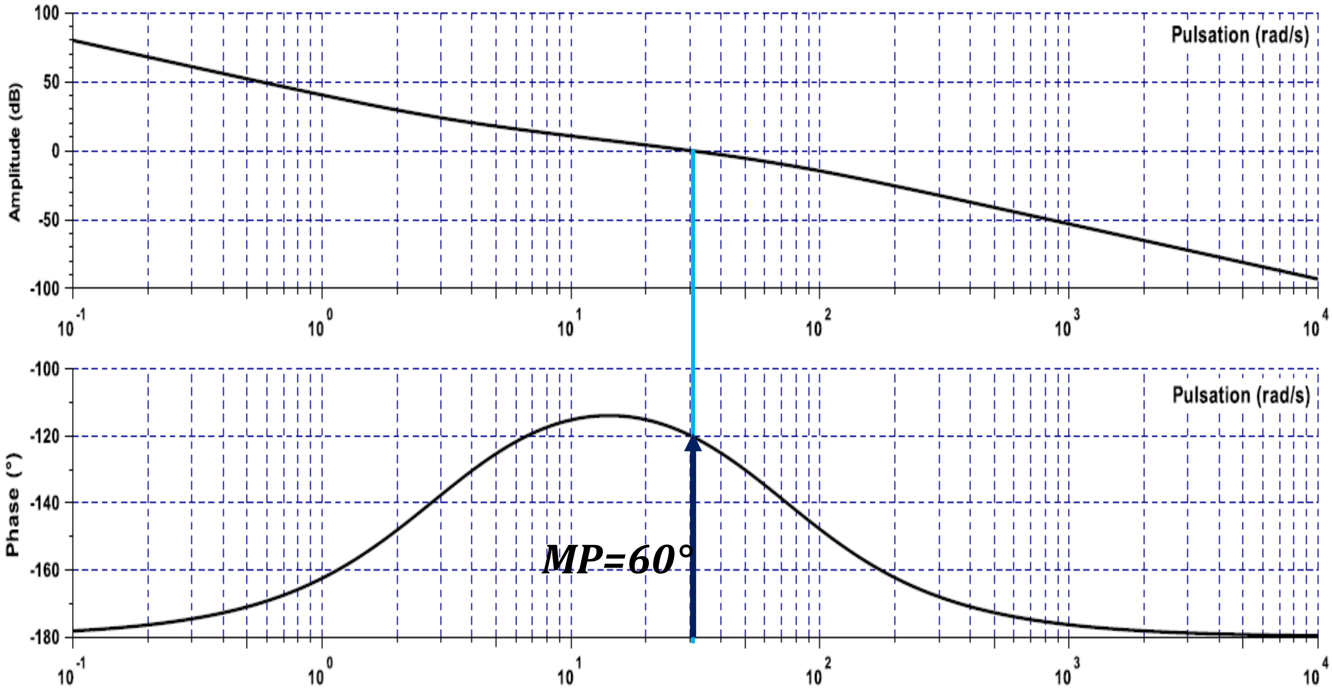
$\Rightarrow \arctan(T_i.\omega_C) - \arctan(T.\omega_C) = 60° \Rightarrow \omega_C = \dfrac{1}{T}\tan\left(\arctan(T_i.\omega_C) - 60°\right)$

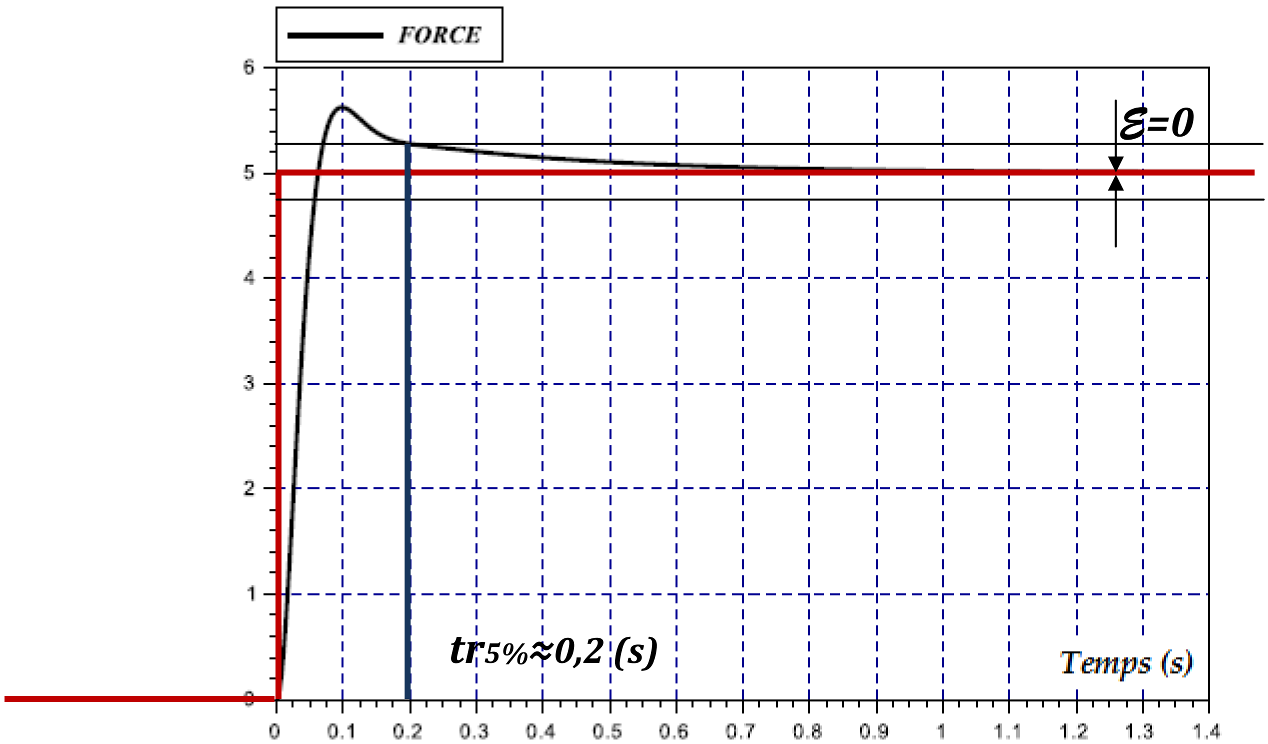
$\omega_C = 30{,}08\,(rad/s)$ $\qquad T_i = \dfrac{10}{\omega_C} = 0{,}332\,(s)$

**b**°- $\left\|H_{BO}(j\omega_C)\right\| = C.\dfrac{K_1}{T_i.\omega_C}.\dfrac{1}{\omega_C}.\sqrt{\dfrac{1+(T_i.\omega_C)^2}{1+(T.\omega_C)^2}} = 1$

$C = \dfrac{T_i.\omega_C}{K_1}.\omega_C.\sqrt{\dfrac{1+(T.\omega_C)^2}{1+(T_i.\omega_C)^2}} = \dfrac{10}{2{,}15}.30{,}08.\sqrt{\dfrac{1{,}2035}{101}} = 15{,}27$ $\qquad C = 15{,}27$

*<u>Question 28 :</u>*




*Stabilité respectée*

*Précision respectée*

*Rapidité respectée*

*Amortissement : dépassement non respecté*

<u>Question 29 :</u>

*L'écart en régime permanent.*

……… ***Ɛ=0***

*<u>Précision respectée</u>*

*Le temps de réponse à 5%.*

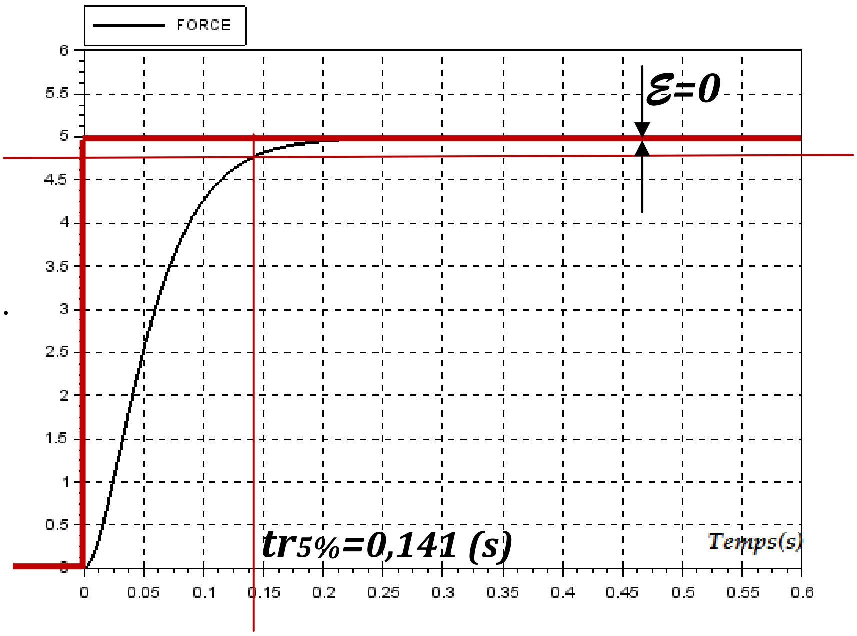
………… ***$tr_{5\%}$=0,141 (s)***……………

*<u>Rapidité respecté</u>*

*Le dépassement.*

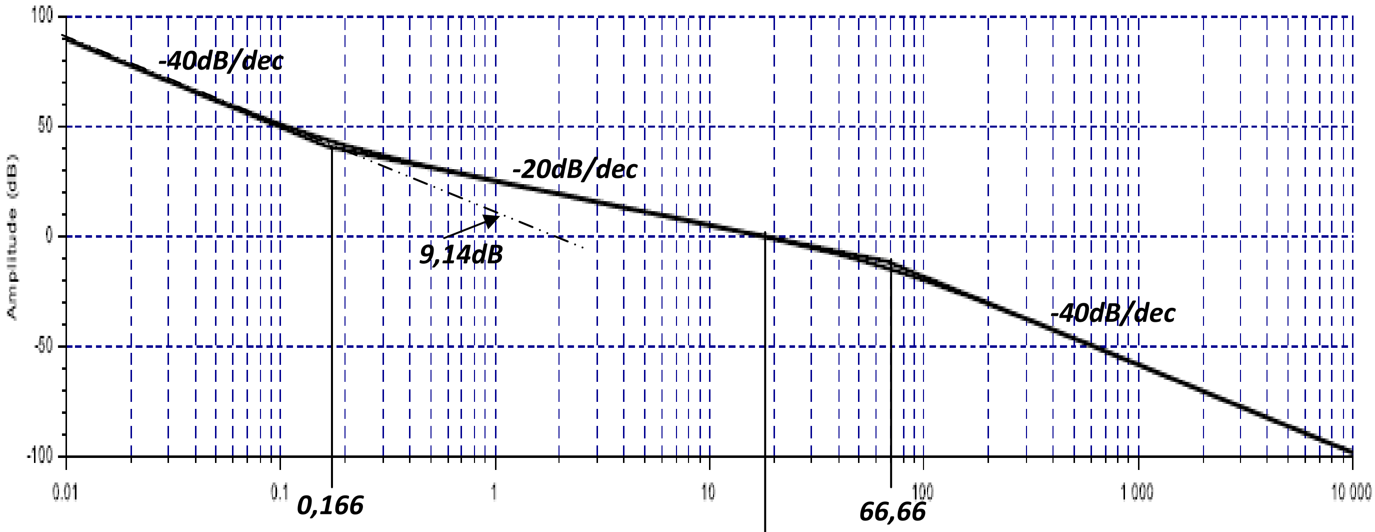
*AUCUN dépassement*

*<u>Dépassement respecté</u>*




<u>Question 30 :</u>

$$H_{BO}(p) = C(p).H_1(p) = C.\frac{1+T_i.p}{T_i.p}.\frac{K_1}{p.(1+T.p)} \quad H_{BO}(p) = 2,86.\frac{1+6.p}{p^2.(1+0,015.p)}$$




*<u>Stabilité respectée</u>*